

# LERC Byte Stream Specification

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## General Remarks about Supported Data Structures

### Basic 2D Image

The basic data structure is the 2D image with image width and height, and a pixel or data type from byte to double. Lerc is often used as a tile format for image container formats such as TIFF, MRF (both in GDAL), or CRF (an ESRI raster format, still proprietary) which can support very large images. Tile sizes used can be as small as 128 x 128 or as large as 2048 x 2048 or more. Tile sizes are often power of 2, but they don't need to be. Anything > 0 is allowed. 1 x 257 would be ok as well.

### 2D Image with an array of values per pixel, or 3D Image

Starting with Lerc codec version 2.4 (or shorter v2.4) an array per pixel is supported as well. This can be used to store data interleaved, such as [RGB, RGB, ...]. Or to represent complex numbers as [Re-Im, Re-Im, ...]. Other use cases are hyperspectral satellite images with order of 100 values per pixel. Or multi-dimensional data. At first we used the term "nDim" for the number of values per pixel. But this turned out to be confusing, so we renamed it to "nDepth" with the release of Lerc codec v2.6. With this Lerc supports a 2D or 3D image or data cube.

### Multiple Bands

Satellite images often have multiple bands. Such bands can be Red, Green, Blue, Infrared. But these bands can also contain data of a different nature, such as a quality band, or a cloud mask. Image container formats such as TIFF store the tiles for different bands often separately. This way they can be accessed independently. If they use Lerc to encode such data, they call Lerc encode for each tile and band separately.

The Lerc API also allows to stack the tiles for multiple bands together into one single compressed byte blob. Since Lerc codec v2.6 each single band header has a field that contains the number of bands or blobs still to come in the byte stream. Together with the blob size information in the single blob header the reader or decoder can jump in memory from header to header to get all information without having to decode the data first. This way Lerc can support 4D images or data cubes.

## Invalid Pixels

Sometimes pixels or image areas need to be marked as invalid, “to be ignored”, or “do not process”. There are mainly 3 different approaches for this: 1) Use a byte or bit mask with each element saying valid or invalid. 2) Use a noData value. Each pixel value that equals this noData value is invalid. 3) Use NaN to mark a pixel value as invalid. This is limited to floating point data.

As Lerc tries to compress the data with a controlled loss in precision and uses math to do that, the mask is the preferred way to mark pixels as valid or invalid. The Lerc API takes one byte mask per band. Byte value 1 means the pixel is valid, 0 invalid. In case of an array per pixel or  $nDepth > 1$ , 0 means all values at that pixel are invalid. 1 means at least one value is valid. The mixed case of valid and invalid values at the same pixel is treated using a noData value. More on that further below. The Lerc API allows to pass one 2D byte mask per band, or one 2D byte mask that is the same for all bands. Or no 2D byte mask at all saying all pixels are valid (unless they are marked invalid by a noData value).

On the Lerc codec level, these byte masks get converted into bit masks and get compressed this way without loss. They usually take very little space in the compressed Lerc blob compared to the compressed pixel values. The Lerc band blob header stores the number of valid pixels, next to image width and height. If all or 0 pixels are valid, no byte mask is stored. If multiple band blobs get concatenated, the first band blob stores the mask if needed. Each consecutive band blob only stores the byte mask if different from the previous band blob. This makes sense as in most cases the mask is the same for all bands.

## NoData value

Up to Lerc codec v2.5 Lerc did not use a noData value. This changed with Lerc codec v2.6. We need to support the mixed case of valid and invalid values at the same pixel. This is a special and somewhat exotic case. Instead of extending the byte mask structure to cover this special case (which would affect everyone who uses Lerc), we went with a noData value. Meaning most users can ignore this and continue using the Lerc API and the byte masks the same way as before. For those users dealing with this mixed case, the Lerc API was extended to new `*_4D()` functions for encode and decode. Those functions allow to pass one noData value per band. On encode, Lerc goes through the pixel data once and pushes such noData values to the byte mask whenever possible. If there are any noData values left, Lerc checks that the noData value is far enough away from the valid data range. If too close or even inside, meaning a valid pixel value might get mapped onto the noData value due to lossy encoding, Lerc

will fall back to lossless encoding to avoid errors. For the noData values itself, Lerc remaps them internally as needed to ensure they do not get modified due to lossy encoding. On decode, they get mapped back to the original noData values.

Note on decode you will only get noData values if they are really used. If they could all be pushed to the byte mask, they will be gone. Thanks to this property, the standard Lerc API decode function will still work for such a Lerc blob. Only if the Lerc blob has a noData value (only for mixed case, nDepth > 1), the standard Lerc API decode function will fail with `ErrCode::HasNoData`. Then the Lerc `decode_4D()` function must be used.

## NaN

Similar to noData, the use of NaN is not encouraged but allowed. Same as with the noData value, Lerc will try to filter out NaN values and push them to the byte mask if possible. If NaN values are left (again, only for mixed case, nDepth > 1), Lerc replaces them by the passed noData value. If no noData value was passed, Lerc encode will fail with `ErrCode::NaN`.

Note on decode you won't get any NaN values. They will be gone.

## LERC Blob per Band

From here on we describe how the data is stored in a single band Lerc memory blob.

Blob Header
Mask
Data Ranges
Pixel Values

Latest Lerc codec: v2.6.

Endianness: Little Endian

Remark: This is the structure in memory. The encoding is done into a memory blob, which can then be saved to disk or transmitted over the network etc.

Multiple values per pixel: Starting with Lerc codec v2.4, multiple (or nDepth) values per pixel can be encoded.

Multiple bands: Here we describe the encoding of one band. For multiple bands the different single band blobs get concatenated.

As an example, an RGB image can now be encoded as a single band blob using nDepth = 3, or band sequential as 3 bands and nDepth = 1.

## LERC Blob Header

Item	Format	Size
Format Identifier String	Char[6] = "Lerc2 "	6
Format Version (= 6)	Int	4
Checksum	UInt	4
Image Height in pixel	Int	4
Image Width in pixel	Int	4
Number of values per pixel	Int	4
Number of valid pixels	Int	4
Micro block size (e.g., 8)	Int	4
Blob size in bytes	Int	4
Lerc datatype	Int	4
Number of more Lerc blobs	Int	4
Uses noData value	Byte	1
Double values are all Integer	Byte	1
Reserved	Byte	1
Reserved	Byte	1
MaxZError	Double	8
zMin = min image value	Double	8
zMax = max image value	Double	8
Internal noData value	Double	8
Original noData value	Double	8

The *Lerc format or codec version number 6* stands for v2.6. This is somewhat historical, there was once a version v1.x. This codec version is different from the official Lerc version number, currently 3.1. This overall Lerc version number can get incremented due to an update to the Lerc API, for instance, without any change to the Lerc codec or codec version number.

*Number of values per pixel* equals the API parameter nDepth.

The following parameters were added in Lerc codec v2.6: *Number of more Lerc blobs* says how many more Lerc blobs follow behind this one, or how many more bands follow in the byte stream. *Uses noData value* set to 1 says that a noData value is used in the data section, and that the noData value must be honored after decode by the caller. 0 means that no noData value is used, and that all invalid pixels, if any, are represented by the valid / invalid bit mask. *Double values are all Integer* is set to 1, not 0, if the data passed as double is all integer or big integer > 32 bit. This is used to promote MaxZError, e.g., from 0 to 0.5. The *Original noData value* is the noData value passed to Lerc encode as used by the caller. The *Internal noData value* is the noData value used by Lerc encode and decode internally. If it differs from the original one, it gets remapped to the original noData value after the core Lerc decode call, and before the decoded data gets handed back to the caller by the Lerc API.

## Bit Mask

The valid / invalid bits are represented as a bit stuffed byte array with 8 bits per byte representing 8 pixels. The size of this byte array is then  $(\text{width} * \text{height} + 7) / 8$ . This bit mask is only encoded if needed, meaning  $0 < \text{number of valid pixels} < \text{width} * \text{height}$ . If all pixels are valid or all invalid the mask is not encoded. The first element is an integer having the size of the mask blob in bytes. If the mask is not encoded, it is 0.

Item	Format	Size
Mask blob size in bytes	Int	4

The bit mask byte array gets run length encoded as follows: If 5 or more consecutive bytes have the same value (255 or 0 being the most common), a negative short integer

equal to the number of same values plus this byte value is encoded (2 + 1 bytes). If less than 5 consecutive bytes have the same value, then a positive short integer equal to the number of different (or not same stretch) bytes is encoded, together with the array of different bytes (2 + n bytes). We use the short integer -32768 to mark the end of the encoded mask blob.

Note that the bit mask, if it gets encoded at all, takes usually much less space than the data itself. Typically around one or a few percent of the space needed for the data. Therefore we try to keep the mask encoding as simple as possible. A more complicated encoding would hardly pay off in terms of better overall compression.

If all valid pixels have the same value ( $zMax = zMin$ ), or all pixels are invalid, then the encoding ends here.

## Data Ranges

Starting with Lerc v2.4, nDepth values per pixel can be encoded. This requires us to save the data ranges as nDepth pairs of min and max values. This is stored binary raw using the pixel data type. In case of nDepth = 4 and data type unsigned short, this would take 16 bytes. In addition, the overall data range is still stored in the header fields zMin and zMax.

## Pixel Values

From here on, the valid / invalid pixels mask is known. Both in encoding and decoding. For encoding, there are 4 possible modes

- Main or block mode. Divide the image into 8x8 or 16x16 blocks (micro block size, see blob header). For each such block, determine block statistics. Based on this block statistics, compress the block. Each block can be compressed and encoded differently.
- Lossless Huffman mode for the 8 bit data types byte or char ( $MaxZError < 1$ ).
- Raw binary mode, write valid data raw binary in one sweep.

- Lossless Huffman mode for data types float or double (MaxZError = 0). Added in Lerc codec v2.6.

The encoder should always pick the mode resulting in the best compression possible. The decoder should assume as little as possible. For instance, the decoder should work for any micro block size, even though only 8 and 16 are currently used.

We go now through these 4 modes, from easier to more complex, starting with the raw binary mode.

## 1. Raw Binary Mode

This mode might be called if only few pixels are valid, such as just one corner of the image. If a large raster is tiled up, then the lower right corner tile could be such a tile with only a small fraction of the pixels valid. In that case it might be best to simply write out the few valid pixels as raw binary. The only compression, if any, is that only valid pixel values are encoded. The bit mask above has the information which pixels are valid and which are not. In the byte stream, we have a single byte flag for this mode:

Item	Format	Size
Flag for write data raw binary in one sweep (1 = yes, 0 = no)	Byte	1

If this byte is 1, all valid pixels (numValidPixels is in the Lerc blob header) are written out raw binary taking the number of bytes (numValidPixels \* nDepth \* sizeof(data type)).

If this byte is 0, the stream continues with one of the other modes.

## 2. Lossless Huffman Mode for 8 bit integer data types

The main mode of Lerc described further below was originally designed for higher bit depth data, such as 16 bit integer or floating point data. It is not optimal for 8 bit data. For this reason this Huffman mode was added later for the important case of encoding 8 bit data lossless. If the bit depth is more than 8 bit, or if the encoding is not lossless (MaxZError >= 1), then the Huffman mode is currently not used for integer data.

In Lerc codec v2.4, the Huffman mode has been extended to  $nDepth > 1$  in two ways. First, the same procedure as for  $nDepth = 1$  is repeated for each array index. However, we still use just one Huffman code book for all the data. Second, as it comes at no extra computational cost, we not only compute the number of bytes needed for Huffman encoding the delta image, but also for Huffman encoding the pixel values directly. And then pick the better one. Here we continue with explaining in more detail the Huffman encoding of the delta image.

The delta image is Huffman encoded as follows. First, the image is turned into a difference or delta image, to make use of the strong local correlation usually present in 2D images. For each pixel, if the left neighbor pixel exists and is valid, subtract its value. If that is not the case, try with the pixel above. If that is not available, subtract the last known valid pixel value. The last known valid pixel value gets initialized with 0.

Second, we compute the histogram over these delta values. When computing the deltas, we make use of the overflow properties of the byte and char data types. For example,  $(\text{Byte})2 - (\text{Byte})255 = (\text{Byte})3$ . Because on decode  $(\text{Byte})255 + (\text{Byte})3 = (\text{Byte})2$ . This way the size of the delta histogram is only 256, not 512. The delta histogram has usually a strong peak at or close to 0.

Third, from this histogram, we compute the optimal Huffman code book. Both histogram and Huffman code book are computed once for the entire image. From this the number of bytes needed for Huffman encoding the image can be directly calculated without actually doing the encoding. This way the encoder can compare to the other modes and decide which one is the best to use.

Item	Format	Size
Image Encode Mode, enum: Block, DeltaHuffman, Huffman	Byte	1

If the Huffman mode wins, the byte stream continues as follows.

Item	Format	Size
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Huffman Format Version (= 4)	Int	4
Huffman blob size in bytes	Int	4
Min Index for non-zero codes (e.g., 217)	Int	4
Max Index for non-zero codes, exclusive (e.g., 321)	Int	4

As the delta histogram often contains a long stretch of zeros, we store the min and max indexes for which we have non-zero Huffman codes. If the histogram has entries for all indexes, then this index range is simply [0, 256]. If we have a peak around 0, then this range could be [217, 321], using wrap-around. The code for index 320 would be the code for index  $(320 - 256) = 64$ . After this header, there are three data sections:

- For the range of non-zero codes, encode the code lengths, index by index or symbol by symbol, bit stuffed at a fixed bit length which is determined by the largest code length. As the max code length allowed is hard coded as 32, the code lengths cannot take more than 6 bits per element which is hardly ever reached.
- For the same range, encode the Huffman codes. As their lengths are already encoded above, we can bit stuff them directly into an array of unsigned integer uint32.
- Encode all valid pixels of the image. For each pixel delta value, bit stuff its variable length Huffman code into an array of unsigned integer uint32. (The encoding of the pixel delta values is the same as for the Huffman codes.)

Let's go through this in more detail. For the first section, we use a more general module for bit stuffing an array of unsigned integer into an array of bytes. The same module is also used by the main or block Lerc mode as described further below. It starts with a small header consisting of 2 numbers. The first one is a byte and contains the number of bits used to encode and bit stuff all elements of the array. As the length of the Huffman code cannot exceed 32 bits (the upper limit set in the encoder), the largest unsigned integer possible to be bit stuffed is 32. This corresponds to length 6. The first 5 bits (0-4) are used to encode this length (here, up to 6). The upper 2 bits (6-7) are used to encode the type of the second number in the header. Depending on the size of the array to be encoded, a uint (0), ushort (1), or byte (2) can be used. Here, with a max number of 256 elements, either 1 or 2 bytes will be needed to encode that number.

Item	Format	Size
Number of bits per element (bits 0-4). Bit 5 = 0 here. Bits 6-7 encode the next type.	Byte	1
Number of fixed length values	UInt (0), UShort (1), or Byte (2)	1 or 2 (or 4, rare)

After this header, the unsigned integer data are bit stuffed using the fixed bit length into a byte array.

The second section is easier to encode, we don't even need a header. We simply bit stuff the Huffman codes in the same order as the Huffman code lengths into an array of uint32. This time the bit length is variable, not fixed. As these code lengths are encoded above, the codes are easy to retrieve.

The third section is the pixel values or data. They get encoded just the same as the Huffman codes. The difference is in the decoding. For the variable length Huffman codes representing the pixel values, the lengths of these codes are not known when decoding them. Lookup tables are used for their decoding.

Remark about encoding the Huffman codes. Since Huffman code book version 4 canonical Huffman codes are used. Meaning they could be constructed by the decoder from the code lengths alone. Despite that, we decided to keep writing and reading the Huffman codes. Reasons:

- It increases the decoder's forward compatibility. We can still change the code without breaking existing decoders.
- It reduces the decoder's complexity. Reading the code is easier than having to construct it. It is easier for a JavaScript or Python developer to write a Lerc decoder.
- The savings from not writing them are small, only a few hundred bytes.

### 3. Main or Block Mode

General remarks, most of which are not mandatory for the byte stream definition:

- Both the image dimensions (width and height) and the pixel is valid / invalid bit mask are known at this point, both in encoding and decoding.

- The image gets divided into square blocks of same size, with 8x8 pixels being the size most often used. This size is encoded in the Lerc header as “micro block size”. The blocks are written left to right, and row by row. Since Lerc codec v2.4 the input image can have nDepth values per pixel. The encoder turns this into nDepth blocks.
- The last block (or last set of nDepth blocks) in a row or column can have different size, down to 1x1 pixels.
- For 8 bit data lossless, both block mode and Huffman mode are tried, and the mode giving better compression is taken. A byte flag is written indicating this choice.
- When the block mode is tried, the data is usually not encoded. Instead block statistics is collected to calculate the compressed block size in bytes.
- After the total number of bytes for block size 8x8 pixels is known, and if the resulting bpp  $\leq 2$ , block size 16x16 pixels is also tried, to reduce the block header overhead.

Encoding one block of N x N pixels:

- Find min and max values zMin and zMax. Count number of valid pixels. Count how many consecutive pixels have the same value.
- Cover special cases: All pixels are invalid, all pixels are 0, or all pixels have the same value z0. In that case, the entire block is encoded as one byte, or as one byte plus offset z0.
- Check if the data can / should be quantized at all. Otherwise encode that block raw binary uncompressed. For instance, if the data type is float, and the value range is large while MaxZError is small (caller asks for high precision), then quantizing the data may not reduce the bpp needed to encode. Here we demand that the max unsigned integer created by the quantization is less than  $2^{30}$ . This is for types of 32 bits or larger, such as int, uint, float, double. For 16 bit types, we demand  $\max < 2^{15}$ , accordingly.
- Write the block header as small as possible. For the offset z0, we write it using a variable data type. For instance, if the data type is integer (int32), but the offset can be written as a short (int16), then it is written as a short. The byte before the offset encodes the data type used in the two upper bits 6-7. For a detailed description of the LERC block header, see below.
- Quantize and bit stuff the data. The pixel values are quantized based on the offset zMin and the error tolerance MaxZError as given by the caller. The valid pixel values get converted into an array of positive integer values (uint32). This array is bit stuffed into a byte array using a fixed bpp based on the max value. An encoding example is given further below.

- If there are enough consecutive pixels having the same value, also try the lookup table (LUT) mode for this block. If there are only a few different (quantized) values present in a block, create an LUT from them, and assign the small index value to each pixel pointing to the correct entry in the LUT. Both the LUT and the array of indexes are then bit stuffed into a byte array using a fixed bpp based on their max values.
- Since Lerc codec v2.5, and for  $nDepth > 1$ , or more than one value per pixel, block  $iDepth$  can now be encoded relative to the previous block ( $iDepth - 1$ ). Example: Assume  $nDepth = 4$ . Before Lerc codec v2.5, 4 blocks of size  $8 \times 8$  pixels were encoded independently (for this block position in the image). This scheme did not exploit the correlation often present in such neighboring bands. Since v2.5, the first block ( $iDepth = 0$ ) gets encoded same as before. For the next block ( $iDepth = 1$ ), the difference to the previous, decoded block is computed. Then the compressed size for both, the raw and the difference or delta block, is computed. The smaller size is taken. So each block  $iDepth$  can get encoded either relative to the previous block ( $iDepth - 1$ ), or absolute same way as before. Which of the 2 it is gets encoded in Bit 2 in the first byte in the Lerc block header. We picked one of the 4 bits which were used before for consistency checking. We keep using the remaining 3 bits for the same consistency check.

## LERC Block Header

Item	Format	Size
<p>Encoding Type (bits 0-1):</p> <p>Value = 0: raw binary</p> <p>Value = 1: quantized and bit stuffed</p> <p>Value = 2: all 0 (encode only this byte)</p> <p>Value = 3: all constant (encode only this header)</p> <p>For cases 0 and 2, the rest of the header is skipped.</p> <p>Before v2.5:</p> <p>Bits 2-5 contain a 4 bit code for the decoder to check the integrity of the data.</p> <p>Since v2.5:</p> <p>Bit 2: 1 means block <math>iDepth</math> is encoded</p>	Byte	1

<p>relative to block (<math>iDepth - 1</math>). 0 means block is encoded as usual, not relative.  Bits 3-5 contain a 3 bit code for the decoder to check the integrity of the data.</p> <p>Bits 6-7 encode the next type.</p>		
Offset	Variable Type	8, 4, 2, or 1

## Variable Type as used in the LERC Block Header

The idea is to use the smallest data type needed to write a data value without loss, and encode that type used in the upper 2 bits 6-7 in the byte right before the variable type entry in the block header.

The 8 bit data types byte and char don't need downsizing, which gets encoded as 0 in the byte before.

The unsigned integer types are also straightforward: ushort (0) tries byte(1). UInt (0) tries ushort (1) and byte (2).

For the signed integer types, we chose: short (0) tries byte (1) and char (2). Int (0) tries ushort (1), short (2), and byte (3).

For the floating point types float and double, we chose: float (0) tries short (1) and byte (2). Double (0) tries float (1), int (2), and short (3).

Data Type Orig / Used	Char	Byte	Short	UShort	Int	UInt	Float	Double
Char	0							
Byte		0						
Short	2	1	0					
UShort		1		0				
Int		3	2	1	0			
UInt		2		1		0		
Float		2	1				0	
Double			3		2		1	0

## Array of unsigned integer (uint32) values bit stuffed using a fixed number of bits per element

Here we describe a core module. It is used to write out the quantized values for each block (in the block mode), or to write out the Huffman code lengths (in the Huffman mode). It starts with a small header:

Item	Format	Size
Number of bits per pixel (bits 0-4).  Bit 5 flags simple bit stuffing (0) or LUT (1) mode.  Bits 6-7 encode the next type.	Byte	1
Number of fixed length values	UInt (0), UShort (1), or Byte (2)	1 or 2 (or 4, rare)

By far the most common case is simple bit stuffing (bit 5 = 0). Here the N different values are bit stuffed into a byte array using a fixed number of bits per pixel, as determined by the largest element in the array.

If the LUT mode offers better compression (bit 5 = 1), then the different array elements are sorted in increasing order. When encoding a LERC block, the min quantized value is always 0 and is therefore omitted in the LUT. The following is written: A byte containing the full size of the LUT (including the leading 0), the LUT bit stuffed using the number of bits per pixel as specified in the above header (without the leading 0), and finally the indexes to the LUT for all valid pixels in the block, also bit stuffed at a fixed number of bits per pixel as determined by the size of the LUT. Note that this bpp must be smaller than the one for the LUT itself, otherwise the LUT mode would not have been chosen.

#### 4. Lossless Huffman Mode for float and double data types

This mode was added in Lerc codec v2.6.

#### Built-in Integrity Checks

We added more rigorous integrity checking starting with Lerc codec v2.3. After encoding the data into a Lerc memory blob, we compute the Fletcher32 checksum starting at the memory address behind the checksum field in the Lerc header and ending with the end

of the blob. The result gets written to the checksum field in the header. Before decoding the Lerc memory blob, we first compute the checksum the same way as above. (We also check that the buffer we got is large enough to hold the Lerc blob based on the blob size stored in the Lerc header.) Then we compare the checksum just computed to the one stored in the header and return false if they differ. The previous consistency checks stay in place as described below.

Wherever possible, intermediate data, such as bit or byte lengths, value ranges, number of elements etc are checked for consistency. Both in the encoder and decoder. If anything inconsistent or out of order gets detected, encoder or decoder return false.

As an extra consistency check, the 4 bits 2-5 in the first byte of the Lerc blob header are filled with the simple “control signature”  $(15 \& (j / 8))$  with  $j$  being the image column coordinate of the upper left pixel of the block being encoded. The decoder checks this code and returns false if it should not match. Since Lerc codec v2.5 only the 3 bits 3-5 are used for this check.

### Example for encoding one block using MaxZError = 0.01

Let’s go through all steps of encoding one 4 x 4 pixels sample data block. Note that blocks are usually larger than that, 8 x 8 or more pixels.

1234.1234	1241.8741	1256.2759	1267.2950
1280.8725	1248.2917	1272.7511	1279.3802
void	1222.2943	1239.3072	void
1264.9720	1250.0852	void	void

Step 1: Calculate the basic statistics for this data block

Min = 1222.2943

Max = 1280.8725

Number of valid pixels = 12



Number of invalid pixels = 4

Step 2: Determine how to encode this block. Assume MaxZError = 0.01.

$$(\text{Max} - \text{Min}) / (2 * \text{MaxZError}) = 2,928.91$$

This is less than  $2^{30}$ . So we can quantize the pixel values from the above block, using

$$n(i) = (\text{unsigned int})((x(i) - \text{Min}) / (2 * \text{MaxZError}) + 0.5)$$

resulting in

591	979	1699	2250
2929	1300	2523	2854
void	0	851	void
2134	1390	void	void

Step 3: Determine number of bits needed and bit stuff these non-negative integers

$$\text{NumBits} = \text{ceil}(\log_2(2929)) = 12$$

Or, in other words,

$$2^{11} < 2929 < 2^{12}$$

So we need 12 bit per number to encode all numbers of this block lossless. There are 12 valid numbers, resulting in  $12 \times 12 = 144$  bit total. As  $144 / 8 = 18$ , we need 18 bytes to encode the entire data block.

Step 4: Write the combined block header (Lerc block header + bit stuffed array header)

Item	Value	Size
Encoding Type (bits 0-1): Bits 6-7 encode the next type	1	1
Offset	1222.2943	4
Number of bits per pixel (bits 0-4):	12	1

Bits 6-7 encode the next type		
Number of valid pixel	12	1

So we need 7 bytes for the block header. The total number of bytes needed for this block is  $18 + 7 = 25$ . Obviously the header takes too much space compared to the raw data. That's why the smallest block size we work with is usually  $8 \times 8$  pixels, not  $4 \times 4$ .

## Example for encoding one block using MaxZError = 1.0

Let's encode the very same pixel block again, but this time using a larger error threshold of 1.0. Step 1, calculate the statistics, is the same as before.

Step 2: Determine how to encode this block. Assume  $\text{MaxZError} = 1.0$ .

$$(\text{Max} - \text{Min}) / (2 * \text{MaxZError}) = 29.29$$

This is less than  $2^{30}$ . So we can quantize the pixel values from the above block, using

$$n(i) = (\text{unsigned int})((x(i) - \text{Min}) / (2 * \text{MaxZError}) + 0.5)$$

resulting in

6	10	17	23
29	13	25	29
void	0	9	void
21	14	void	void

Step 3: Determine number of bits needed and bit stuff these non-negative integers

$$\text{NumBits} = \text{ceil}(\log_2(29)) = 5$$

Or, in other words,

$$2^4 < 29 < 2^5$$

So we need 5 bit per number to encode all numbers of this block lossless. There are 12 valid numbers, resulting in  $5 \times 12 = 60$  bit total. As  $60 / 8 = 7.5$ , we need 8 bytes to encode the entire data block.

Step 4: Write the combined block header (Lerc block header + bit stuffed array header)

Item	Value	Size
Encoding Type (bits 0-1): Bits 6-7 encode the next type	1	1
Offset	1222.2943	4
Number of bits per pixel (bits 0-4): Bits 6-7 encode the next type	5	1
Number of valid pixel	12	1

So we need 7 bytes for the block header. The total number of bytes needed for this block is  $8 + 7 = 15$ .